Norwegian University of Science and Technology
Department of Mathematical Sciences

MA3403 Algebraic Topology Fall 2018

Exercise set 1

1 Let $f\colon X \to Y$ be a continuous map between topological spaces X and Y.

a) Let $K \subseteq X$ be compact. Show that $f(K) \subseteq Y$ is compact.

b) Give an example of a map f and a compact subset $K \subseteq Y$ such that $f^{-1}(K) \subseteq X$ is not compact.

2 Draw a picture of S^2 as a cell complex with six 0-cells, twelve 1-cells and eight 2-cells.

3 Show that the stereographic projection

$$\phi\colon S^1 \to \mathbb{R} \cup \{\infty\},\ (x, y) \mapsto \begin{cases} \frac{x}{1-y} & y \neq 1 \\ \infty & y = 1 \end{cases}$$

defines a homeomorphism from S^1 to the one-point compactification $\hat{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$ of $\mathbb{R}$.

4 **a)** Let X and Y be topological spaces. Show that homotopy defines an equivalence relation on the set $C(X, Y)$ of continuous maps $X \to Y$.

b) Show that *being homotopy equivalent* defines an equivalence relation on topological spaces.

5 **a)** Show that S^1 is a strong deformation retract of $D^2 \setminus \{0\}$.

b) Show that $D^2 \setminus \{0\}$ is not contractible.