Math 231br
Problem Set 5

Spring 2014

You should hand in solutions to at least three problems. For any questions, please send me an email to gquick@math.harvard.edu and/or come to my office hours on Wednesdays 1.30-2.30pm in SC 341. Solutions are due by Friday, April 11, at the beginning of class.

Problem 5.1. For $2n$-spheres, the Adams operations act as

$$\psi^k(x) = k^n x \text{ for } x \in \tilde{K}(S^{2n}).$$

Problem 5.2. Let X be a pointed compact Hausdorff space and let β be the periodicity isomorphism. Decide by computing both comopositions if the following diagram commutes:

$$\begin{array}{ccc} \tilde{K}(X) & \xrightarrow{\beta} & \tilde{K}(S^2 \wedge X) \\ {\scriptstyle \psi^k}\downarrow & & \downarrow{\scriptstyle \psi^k} \\ \tilde{K}(X) & \xrightarrow[\beta]{} & \tilde{K}(S^2 \wedge X). \end{array}$$

Can we extend the ψ^k to operations on the $\mathbb{Z}$-graded theory $K^*(X)$?

Problem 5.3. Prove Theorem 25.6 of Lecture 25:
Given a vector bundle $E \to X$ over a compact Hausdorff space X, there is a compact Hausdorff space $F(E)$ and a map $p\colon F(E) \to X$ such that the induced map $p^*\colon K^*(X) \to K^*(F(E))$ is injective and $p^*(E)$ splits as a sum of line bundles. (Hint: Use the projective bundle formula for K-theory.)

Problem 5.4. Prove Lemma 25.3 of Lecture 25:
The polynomial Q_k with $Q_k(\sigma_1, \ldots, \sigma_k) = x_1^k + \cdots + x_n^k$ satisfies the recursive formula

$$Q_k = \sigma_1 Q_{k-1} - \sigma_2 Q_{k-2} + \cdots + (-1)^{k-2}\sigma_{k-1}Q_1 + (-1)^{k-1}k\sigma_k.$$